1960

The determination of the ratio of the coefficients of eddy diffusivity for heat and momentum from micrometeorological data.

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Monterey, California. Naval Postgraduate School

http://hdl.handle.net/10945/11981

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
METEOROLOGY

United States Naval Postgraduate School
Monterey, California

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This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

METEOROLOGY

from the

United States Naval Postgraduate School

ABSTRACT

This work describes a method of determining the ratio of heat and momentum diffusivities from micrometeorological data obtained by the Great Plains Turbulence Field Program [7].

The method involves obtaining first the value of the Deacon wind profile parameter, used as the indicator of stability, and with this value, the ratio of two forms of the Richardson number, which is theoretically equivalent to the ratio of heat and momentum diffusivities.

This investigation was carried out at the U. S. Naval Postgraduate School, Monterey, California, during the period February 1959 - May 1959, in partial fulfillment of the requirements for the degree of Master of Science in Meteorology.

The writer wishes to express his appreciation for the assistance and guidance given him by Professor Frank L. Martin in this investigation.

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TABLE OF SYMBOLS AND ABBREVIATIONS

Symbols:

°C	degrees Centigrade
Ri	Richardson's number
Rf	flux Richardson number
T	temperature
g	acceleration due to gravity
k	von Karman's constant
l	mixing length
ln	natural logarithim
cm	centimeters
m	meters
u	horizontal wind component
U	horizontal wind component
u_*	friction velocity
w^*	vertical mixing velocity
x	a form of Richardson's number
X	a form of flux Richardson's number
z	height
z_0	roughness parameter
β	Deacons wind profile parameter
θ	potential temperature
τ	eddy shearing stress
ρ	density of air
A	exchange coefficient
N_θ	the ratio of eddy diffusivities for heat and momentum

a	adiabatic
m	mean for layer considered at a given time
numbers	height or layer considered
bar	weighted average of all layers at a given time
$=$	equal to
$\leqslant$	less than or equal to
$\lvert \; \rvert$	absolute value
$\frac{\partial}{\partial z}$	partial derivative with respect to height
Δ	finite difference
Σ	summation of all values from 1 to N

1. Introduction

A theory is introduced, after Lettau [6], and Martin [8], which permits computation of special forms of the Richardson number and flux Richardson number. These equations are dependent upon the Deacon wind profile parameter β, and the roughness parameter z_o. Equations for the computation of these parameters are derived also.

A recent theory is included in these determinations, and in a sense, this paper constitutes an attempt at testing this theory and comparing its results to that of the many previous investigators in the field of atmospheric turbulence.

The data utilized in this study is evaluated and is considered to overcome many of the considerable limitations inherent in data available for earlier research.

Correlation coefficients of the resulting ratio of heat and momentum diffusivities with the two forms of the Richardson number and the Deacon wind profile parameter were computed, and the dependence of the ratio upon the other variables is discussed and compared in the light of knowledge gained by earlier investigators.

2. Theory and derivations

The work of von Karman, Prandtl and Nikuradse has shown that the fundamental relationship for the wind shear within a neutral surface layer is given by [2],

$$\frac{\partial u}{\partial z} = \frac{u_{*a}}{kz} \qquad (1)$$

where u = the mean speed at a height z above the surface

u_{*a} = $\sqrt{\tau_a/\rho}$ called the "friction velocity" in the neutral layer

τ_a = the shearing stress at height z

ρ = fluid density

k = von Karman's constant

On integration, Eq. (1) gives the logarithmic law for the neutral surface layer (defined as the layer in which the shearing stress is effectively constant),

$$u_a = \frac{u_{*a}}{k} \ln \frac{z}{z_o} \qquad (2)$$

which applies to fully rough flow, and z_o, the roughness parameter, is the measure of surface roughness.

In studying the wind profiles versus the logarithm of height as ordinate under non-neutral conditions, Deacon [2] noted that the wind profile differed from the logarithmic profile in that it showed convex curvature under unstable conditions, and reverse curvature under stable conditions. He has suggested that the relationship,

$$\frac{\partial u}{\partial z} = \frac{u_*}{k z_o^{1-\beta}} z^{-\beta} \qquad (3)$$

should exist under non-neutral conditions, where β is the wind profile curvature parameter.

Applying the condition that u = 0 at $z = z_o$, which holds for the logarithmic profile, and integrating, this expression becomes,

$$u = \frac{u_*}{k(1-\beta)}\left\{\left(\frac{z}{z_o}\right)^{1-\beta} - 1\right\} \qquad (4)$$

Assuming that $\frac{u_*}{k(1-\beta)}$ remains constant in the layer z_1 to z_2, one may write (2),

$$\frac{u_2}{u_1} = \left\{\frac{(z_2/z_o)^{1-\beta} - 1}{(z_1/z_o)^{1-\beta} - 1}\right\} \qquad (5)$$

Determination of z_o using an atmosphere of neutral stability allows one to calculate β knowing the winds at the heights 1 and 2. Eq. (5), with $\frac{u_2}{u_1}$ regarded as known, may be solved for β by a method of successive approximations known as the Newton-Raphson method. In this method, successive approximations to a root of any equation f(y)=0 may be written in iterative form as,

$$y_{i+1} = y_i - \frac{f(y_i)}{f'(y_i)} \qquad (6)$$

Rearranging Eq. (5) into the form f(y)=0, where $y = 1 - \beta$, one obtains,

$$f(y) = \frac{u_2}{u_1}\left(\frac{z_1}{z_o}\right)^{y} - \frac{u_2}{u_1} - \left(\frac{z_2}{z_o}\right)^{y} + 1 = 0 \qquad (7)$$

Placing the above expression into the iterative form, and making use of the identity, $\log_e\left(\frac{z_1}{z_o}\right) = \log_{10}\left(\frac{z_1}{z_o}\right)\log_e 10$

gives,

$$y_{i+1} = y_i - \frac{\left[\frac{u_2}{u_1}\left(\frac{z_1}{z_o}\right)^{y_i} - \frac{u_2}{u_1} - \left(\frac{z_2}{z_o}\right)^{y_i} + 1\right]}{\left[\frac{u_2}{u_1}\left(\frac{z_1}{z_o}\right)^{y_i}\log_{10}\left(\frac{z_1}{z_o}\right) - \left(\frac{z_2}{z_o}\right)^{y_i}\log_{10}\left(\frac{z_2}{z_o}\right)\right]\log_e 10} \qquad (8)$$

This method of obtaining β was used in this paper and requires only several iterations to obtain the accuracy specified in Section 4.

Since the solution for β from Eq. (8) is dependent upon the roughness parameter, a method of determining its value under the applicable conditions must be devised. As indicated, z_0 values may be determined under neutral conditions at sunrise and sunset. Since z_0 is independent of lapse rate [3], use is made of the following dimensionless ratio, due to Davidson and Barad [1],

$$R = \frac{[(u_4 - u_1) + (u_3 - u_2)]}{[(u_4 + u_3 + u_2 + u_1)]} = \left(\frac{\Delta u}{\Sigma u}\right) \qquad (9)$$

into which the expression for the logarithmic profile may be substituted. This gives

$$\frac{\Delta u}{\Sigma u} = \frac{\ln z_4 + \ln z_3 - \ln z_2 - \ln z_1}{\ln z_4 + \ln z_3 + \ln z_2 + \ln z_1 - 4 \ln z_0} \qquad (10)$$

Upon further reduction of Eq. (10) an expression for z_0 is obtained,

$$z_0 = \ln^{-1}\left[\frac{\Sigma \ln z_i}{4} - \frac{\Delta \ln z}{4\,\frac{\Delta u}{\Sigma u}}\right] \qquad (11)$$

where $\Sigma \ln z_i = \ln z_4 + \ln z_3 + \ln z_2 + \ln z_1$, $\Delta \ln z = \ln z_4 + \ln z_3 - \ln z_2 - \ln z_1$

a. Turbulence theory

The derivation of the remainder of the theory in this paper is dependent upon Lettau's turbulence model [6], which was adapted to the data in a manner similar to that of Martin [8].

In analogy to molecular diffusion, the theory of turbulent mixing defines the exchange coefficient as,

$$A = -\rho \overline{l' w'} \qquad (12)$$

which is written by Lettau [6] as,

$$A = \rho l \overset{*}{w} \qquad (13)$$

where $\overset{*}{w}$ is called the mixing velocity and l the root-mean-square mixing length introduced by Prandtl [3]. Since turbulence theory also states that,

$$\tau = -\rho \overline{u'w'} = A(\partial u/\partial z) \qquad (14)$$

one may write that,

$$\tau = \rho \overset{*}{u} \overset{*}{w} \qquad (15)$$

where $\overset{*}{u}$ is called the virtual friction velocity.

Combining the last three equations we obtain,

$$\overset{*}{u} = l(\partial u/\partial z) \qquad (16)$$

Lettau [6] assumes under adiabatic conditions, that

$$u_{*a} = \overset{*}{u}_a = \overset{*}{w}_a \qquad (17)$$

Under non-adiabatic conditions, Lettau [6] makes the following assumptions,

$$\frac{\overset{*}{w}}{l} = \frac{\overset{*}{w}_a}{l_a} \qquad (18)$$

$$\frac{\overset{*}{w}^2}{l} = \frac{\overset{*}{w}_a^2}{l_a} - N_\theta \frac{g\,l}{\theta_m} \frac{\partial \theta}{\partial z} \qquad (19)$$

where $N_\theta = K_H/K_M$, the ratio of the coefficient of eddy diffusivity for heat to that of eddy diffusivity for momentum.

Substituting the expression for $\overset{*}{w}$ from Eq. (18) into Eq. (19), gives upon rearrangement,

$$1 = \frac{l_a}{l} - N_\theta \frac{g\, l_a^2}{\overset{*}{w}_a^2 \theta_m} \frac{\partial \theta}{\partial z} \qquad (20)$$

It should be noted at this point, that in the last term of Eq. (20), the term N_θ represents the efficiency in the conversion of thermal energy into kinetic energy. The last term of Eq. (20), which will be denoted x, may be written

$$x = N_\Theta X \,, \quad X = \frac{g\, l_a^2}{\overset{*}{w}_a^{2}\, \Theta_m} \frac{\partial \Theta}{\partial z} \tag{21}$$

where X is a particular form of the Richardson number introduced by Lettau [6]. Actually,

$$X = Ri \left(\frac{\partial u / \partial z}{\partial u_a / \partial z} \right)^2 \tag{22}$$

but X will henceforth be called the "Richardson Number", whereas x will be called the flux Richardson number [6].

Combining Eqs. (18), and (19), Lettau [6] obtains,

$$l = \frac{l_a}{(1+x)} \tag{23}$$

Combining Eqs. (3), (15), (18), and (23), enables one to derive the following results,

$$u_* = \frac{\overset{*}{w}_a}{(1+x)^2} \left(\frac{z}{z_c} \right)^{1-\beta} \tag{24}$$

$$\frac{\partial u}{\partial z} = \frac{\overset{*}{w}_a}{k z_c (1+x)^2} \left(\frac{z}{z_c} \right)^{1-2\beta} \tag{25}$$

This derivation is shown by Martin [8].

(b) Use of finite-difference derivatives

In computations with Eq. (25), it is necessary to use finite-difference derivatives. The instruments were arranged at constant logarithmic increments so that the following

transformations due to Lettau [7] are of value,

$$U_u' = \frac{\partial U_a}{z\,\partial \ln z}$$

$$n^2 = z_2/z_1 \qquad (26)$$

$$z_{1.2} = (z_1 z_2)^{1/2}$$

where $z_{1.2}$ represents the geometric mean of any layer z_1 to z_2, and $n^2=2$, for the data levels utilized in this work. Therefore,

$$z_{1.2} = (n^2 z_1^2)^{1/2} = \sqrt{2}\, z_1 \qquad (27)$$

and, in terms of finite-differences, $\partial u/\partial z$ becomes,

$$\frac{\partial u}{\partial z} = \frac{u_2 - u_1}{z \ln z_2/z_1} = \frac{u_2 - u_1}{z \ln n^2} = \frac{u_2 - u_1}{z \ln 2} \qquad (28)$$

Letting $z=z_{1.2}$ makes Eq. (28) representative at a mean height in the layer z_1 to z_2, and $\partial u/\partial z$ becomes,

$$\frac{u_2 - u_1}{z_{1.2} \ln 2} = \frac{\overset{*}{w}_a}{k z_0 (1+x)^2} \left(\frac{z_{1.2}}{z_0}\right)^{1-2\beta} \qquad (29)$$

Rearranging leads to

$$(1+x)^2 = \frac{\overset{*}{w}_a \ln 2}{(u_2 - u_1) k} \left(\frac{z_{1.2}}{z_0}\right)^{1-2\beta} \qquad (30)$$

and solving for x, gives,

$$x = \left\{ \left[\frac{\overset{*}{w}_a \ln 2}{(u_2 - u_1) k}\right]^{1/2} \left(\frac{z_{1.2}}{z_0}\right)^{1-\beta} - 1 \right\} \qquad (31)$$

Hence, it becomes possible to solve for x, knowing $\overset{*}{w}_a$, β, and z_0.

(c) An expression for $\dfrac{\overset{*}{w}_a}{k}$

Strictly speaking $\overset{*}{w}_a$ varies with stability and with geostrophic wind. The variation with stability is not yet accurately known but the variation from one adiabatic time to

another may be assumed to be linear. An expression for $\overset{*}{w}_a$ in terms of the neutral-wind profile may be obtained by logarithmic differentiation of Eq. (2), with u_{*a} replaced by $\overset{*}{w}a$, giving,

$$\frac{\partial U_a}{\partial \ln z} = \frac{\overset{*}{w}_a}{k} \qquad (32)$$

which, on expansion, becomes

$$\frac{\overset{*}{w}_a}{k} = \frac{[(U_{a_4} - U_{a_1}) + (U_{a_3} - U_{a_2})]}{(\ln z_4 - \ln z_1) + (\ln z_3 - \ln z_2)} = \frac{\Delta U_a}{4 \ln 2} \qquad (33)$$

where ΔU_a is notation for

$$U_{a_4} - U_{a_1} + U_{a_3} - U_{a_2} = \Delta U_a$$

and $\ln \frac{z4}{z1} + \ln \frac{z3}{z2} = 4\ln 2$ with instrumentation as set up in the Great Plains project.

Substitution of Eq. (33) into Eq. (31 gives

$$x = \left\{ \left[\frac{\Delta U_a}{4(U_2 - U_1)} \right]^{1/2} \left(\frac{z_{1.2}}{z_0} \right)^{1-\beta} - 1 \right\} \qquad (34)$$

Variation of ΔU_a linearly to coincide with the bi-hourly observations between the neutral times at sunrise and sunset will give representative values of the flux Richardson number x during the day.

(d) A convenient formula for X

The particular form of the Richardson number used in the Lettau theory was shown in Eq. (21). This form, henceforth denoted X, may be written as [7],

$$X = \frac{g\,\Theta'}{\Theta_m\, {}'U_a^{2}} \qquad (35)$$

Similarly to Eqs. (26), we have the transformation

$$z\,\Theta' = \frac{z\,\partial\Theta}{\partial z} = \frac{\partial\Theta}{\partial \ln z} \qquad (36)$$

and therefore in terms of finite-differences, the following relationships hold at $z = z_{1.2}$.

$$\Theta' = \frac{\Theta_2 - \Theta_1}{z_{1.2} \ln n^2}, \quad u_a' = \frac{u_{a2} - u_{a1}}{z_{1.2} \ln n^2} \tag{37}$$

Substituting Eqs. (37) into Eq. (35) gives,

$$X = \frac{g z_{1.2} (\Theta_2 - \Theta_1) \ln n^2}{\Theta_m (u_{a2} - u_{a1})^2} \tag{38}$$

where $u_{a2} - u_{a1}$ may be averaged over all successive pairs of levels to be Δu_a 4 of Eq. (33). In Eq. (38) θ_m may be approximated by $\theta_m \doteq T_2$. Making the substitution $u_{a2} - u_{a1} = \Delta u_a/4$ gives,

$$X = \frac{16 g \ln 2 (\Theta_2 - \Theta_1) z_{1.2}}{T_2 (\Delta u_a)^2} \tag{39}$$

Since $\frac{v_a^*}{k}$ no longer appears in either x or X, but has been replaced by Δu_1, the latter has been varied linearly between times of neutral stability (near sunset or sunrise).

(e) N_θ as a function of height and other variables

In Eq. (21), it was shown that,

$$N_\theta = \frac{x}{X}$$

and we now have convenient working equations for x and X, Eqs. (34), and (39), respectively. Computations of N_θ as a function of elevation can now be made provided the roughness parameter z_0 and the wind profile parameter β have first been obtained.

3. Nature of the data

In addition to testing the micrometeorological theory set forth in this paper, this investigation was also initiated by the availability of comprehensive round-the-clock data of the highest caliber to date. The Great Plains Turbulence Program [7] was a research project of the U. S. Air Force Geophysics Research Directorate conducted in open prairie country near O'Neill, Nebraska between, 1 August, and 10 September 1953.

Fourteen organizations participated in the project, and their contributions included: instrumentation, data collection and evaluation, development of turbulence theory, instrument description and evaluation, site preparation, and data tabulation.

The Great Plains Turbulence Program was in progress for about six weeks during which time seven observation periods were completed. The criteria for an acceptable observation period were that the daytime periods be clear to partly cloudy, with clear skies at night, so that large diurnal variations in the heat budget terms, and particularly, in the Richardson number could be realized. Also, it was preferred that the mean geostrophic wind speed vary from one observation period to the next. Instrument arrangement on the field site required that the general observation periods be characterized by relatively constant wind direction between SE and SW [7] .

Only the data of the sixth and seventh general observation periods, for the days, 31 August through 1 September, and 7-8 September 1953 [7] , respectively, will be described and evaluated

in this paper.

Massachusetts Institute of Technology, and Johns Hopkins University (hereafter referred to as MIT, and JHU, respectively), wind and temperature profile observations at standard levels were utilized. The MIT data collected at the 2, 4, 8, and 16-meter levels consisted of 15-minute mean wind speed (cm/sec), and 15-minute mean air temperatures (°C). JHU data was collected at .4, .8, 1.6, and 3.2-meter levels and consisted of hourly-mean wind speed (cm/sec), and average temperatures from 20 readings during a 5-minute interval. Bi-hourly observations were taken from the time the general observation period began with all time averages centered at observation time.

The standard cup anemometers used by MIT were of the conventional type and were wind-tunnel calibrated prior to installation. Differences in calibration were minimized by field matching tests conducted at the test site. Wind measurements were averaged over 15-minute intervals and recorded to the nearest cm/sec. JHU obtained a number of three cup anemometers of 1941-1942 model from the Soil Conservation Service, and by modification and careful matching produced a set of mechanical wind measuring instruments with compatability of the order of 0.5%. Wind measurements were of hourly mean wind speed recorded to the nearest cm/sec.

MIT temperatures were measured with identical, artificially ventilated thermocouples, similar instrumentation having previously been used at MIT's Round Hill Field Station. The thermojunctions were shielded from solar and ground radiation and ventilated at a rate of 5 m/sec. Calibration was conducted in water baths,

matched against precision mercury thermometers. Fifteen-minute mean temperatures at the standard levels used in this paper were recorded to the nearest 0.01°C.

The description of JHU temperature installation is similar to that of MIT's, the main differences being in the method of shielding and the lack of artificial ventilation. Calibration was similar to MIT procedure previously described. Using Beckmann differential mercury thermometers between the two water baths, an absolute accuracy of 0.05°C was indicated. An average of 20 temperature readings at each level during a 5-minute period was recorded to the nearest 0.01°C. A detailed description of the instrumentation, etc., is given in Lettau [7].

4. Computations

The solution of the working equations for β, z_o, x, and X; equations (8), (11), (34), and (39) respectively, was accomplished by desk calculator, rather than by slide rule or electronic computer. It is felt that use of the desk calculator was justified considering the relatively small quantity of data. Also, greater accuracy than that attainable by slide rule, was assured, to the limits of the data significance.

(a) The roughness parameter, z_o

Determination of z_o by means of a fitted straight line on a ln z versus wind speed graph can afford only a rough approximation, at best. Accordingly, the working equation,

$$z_o = \ln^{-1}\left[\frac{\Sigma \ln z_i}{4} - \frac{\Delta \ln z}{4\frac{\Delta u}{\Sigma u}}\right] \qquad (11)$$

applicable at the adiabatic times of sunrise and sunset, was used. Fortunately, the 0635 and 1835 observation times were near enough to local sunrise and sunset at this time of the year (31 Aug. - 8 Sept. 1953) to satisfy the adiabatic requirement.

For the MIT data levels of 2, 4, 8, and 16 m, Eq. (11) with $z_4 = 2^4$, $z_3 = 2^3$, $z_2 = 2^2$, and $z_1 = 2$ meters may be reduced to the form

$$z_o = \log_{10}^{-1}\left[\left(2.5 - \frac{\Sigma u}{\Delta u}\right)\log_{10} 2\right] \qquad (40)$$

The roughness parameter for the JHU data levels of .4, .8, 1.6, and 3.2 m, is obtained by reducing Eq. (11) to,

$$z_o = \log_{10}^{-1}\left[\left(0.178078 - \frac{\Sigma u}{\Delta u}\right)\log_{10} 2\right] \qquad (41)$$

where again, $\Sigma u = u_4 + u_3 + u_2 + u_1$ and, $\Delta u = u_4 - u_2 + u_3 - u_1$

The above formulas are solved very simply by desk calculator and logarithmic tables. The value of z_0 obtained by this method is made to hold for all computations within a 12 hour period centered on the time for which it is determined.

(b) The iterative procedure for $y_i = 1 - \beta$

Eq. (8) required only that the roughness parameter be known, in addition to the observed data. No suitable objective method for obtaining an initial estimate of y_i could be devised. Accordingly, after some experience in the use of iterative steps, it was found that subjectively-determined starting values for the other bi-hourly times, based on the premise that β increases with diurnal heating and approaches one nearer the ground, provided for more rapid convergence of the computation. It was observed that the solution converged quite nicely for all but near-neutral conditions, where considerable difficulty was encountered. Accordingly, criteria for accuracy of the solution were devised which simplified the computations a great deal. That is, if $|y_i| > .10$, then the solution could be halted when two places of decimals had been fixed. However, for $|y_i| \leq .10$, and if the solution was oscillating over the desired root, it was required that three places of decimals be unchanged after further iterants are obtained. If the solution was not oscillating, but approaching the root from above or below, then four places of decimals were determined without change in a successive iterant. Also, it was decided to disregard the solution if $|y_i| < .005$, for this case was considered identical to $\beta = 1$.

In addition, certain data-series of near neutral stability were excluded: in particular, those cases in which $|\Theta_2-\Theta_1|$ in any layer was less than 0.05°C.

(c) Computation of x

As was pointed out in the discussion following Eq. (31) the solution for x requires the knowledge of z_0, β, and some representation of $\overset{*}{w}_a$ under non-neutral conditions. This representation was accomplished in Eq. (34) by allowing the function $\Delta u_a = (u_{a_4} - u_{a_2}) + (u_{a_3} - u_{a_1})$ to vary linearly between the 0635 and 1835 adiabatic conditions. The value of x obtained is generally negative for unstable cases and positive for stable cases.

The roughness parameter has already been determined for the solution of β in any layer, and is applied in the solution for x in the same manner as before. The value of $z_{1,2}$ appearing over z_0 in Eq. (34) makes the value of x obtained applicable at the geometric mean of the layer concerned, whereas the β value was assumed to be representative of the entire layer z_1 to z_2.

(d) Computation of X

The solution of Eq. (39) for the Richardson number for any layer is quite straightforward. As observation shows, it contains only constants and values already obtained, or from the data. The value for g was taken to be 980 cm/sec^2. To determine the value of $\theta_2 - \theta_1$, the expression $\Theta_2 - \Theta_1 = (T_2 - T_1) + (z_2 - z_1)\gamma_d$ was used, where $\gamma_d = .98\ °C/100\ m$.

As was mentioned earlier, a criterion for determining a non-

neutral layer from the temperature sounding was set up, namely, $|\Theta_2 - \Theta_1| > .05°C$. In the contrary case, the data was not used for computation purposes. The value of X obtained has the same sign as the temperature lapse, negative for unstable conditions, and positive for stable conditions. Since $z_{1.2}$ appears in the equation for X, its solution is also applicable at the geometric mean of the layer. The working equation for X now becomes,

$$X = \frac{16\, g \ln 2 \left[(T_2 - T_1) + (z_2 - z_1)\left(.98/100\right) \right]_{z_{1.2}}}{T_2 (\Delta U_a)^2} \qquad (42)$$

The solution for N_θ is simply the value of x from Eq. (34) divided by the value of X from Eq. (39). The value for N_θ thus obtained is always positive, and applies at the geometric mean of the layer, z_1 to z_2.

5. Discussion of results

This section contains a summary of results for the computations of β, x, X, and N_θ, and compares these results to those obtained by previous investigators. The numerical values of the above parameters are shown in Tables 1, 2, and 3 for the indicated times, and layers.

Deacon [2] quotes values of the wind profile parameter, which he obtained from Eq. (5) over short grass, using wind speeds for heights between the 0.5 and 4-m levels, as ranging from 0.72 to 1.13. Inspection of Tables 1, 2, and 3 shows a range of β of 0.85 to 1.14, a somewhat lesser range than that obtained by Deacon [2] from the series of data he utilized at Porton, England.

Priestley [9], shows a plot of log N_θ versus x, for mainly unstable periods, in which values of Rf range from 0.1 to -0.5. His scatter diagram is shown below as Fig. 1 of this paper.

Based on this data, Priestley obtained an average correlation coefficient between N_θ and $Rf_{1.5}$ of -0.81.

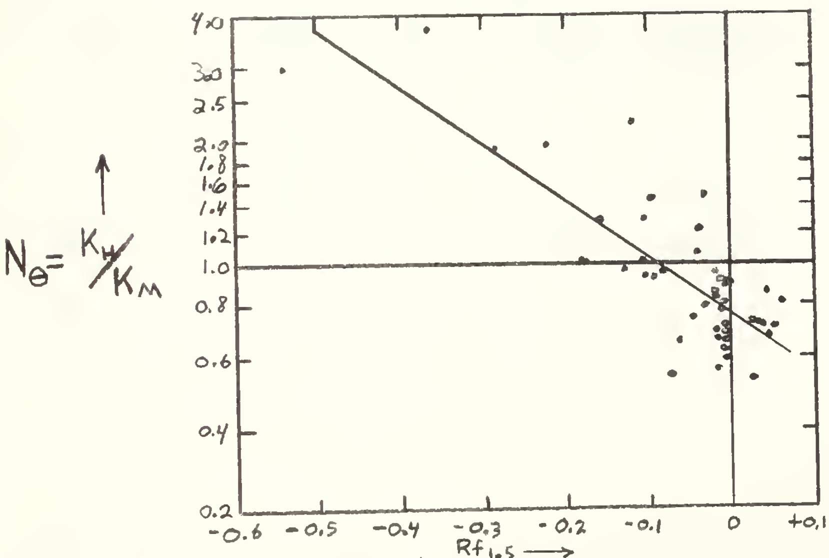

Fig. 1. Dependence of K_H/K_M on Rf at 1.5 meters (after Priestley [9]).

Table 1. Computations from the sixth general observation period (MIT data only).

Layer (m)	Para-meter	Time of observation							
		31 August 1953							
		0435	0835	1035	1235	1435	1635	2035	2235
8-16	β	.9174	1.1274	1.0775	1.1436	1.0224	1.0145	.8692	.9027
	x	.5244	-.4987	-.3647	-.5385	-.2269		1.1614	
	X	.0799	-.0251	-.0860	-.0596	-.0888		.0897	
	N_θ	6.5600		4.2420	9.0340	2.5560		12.9480	
4-8	β	.9922	1.1207	1.1093	1.1270	1.0714	1.0153		.9823
	x	2.0605		-.4303	-.3891	-.3770	-.2949		.1629
	X	.0267		-.0699	-.0738	-.0671	-.0341		.0309
	N_θ	2.2660		6.1540	5.2690	5.6270	8.6550		5.2720
2-4	β		1.0714	1.1173	1.0962	1.0475			
	x		-.3226	-.4257	-.3900	-.2967			
	X		-.0378	-.0745	-.0810	-.0625			
	N_θ		8.5420	5.7100	4.8160	4.7460			
		1 September 1953							
		0035	0235	0435	0635	0835			
8-16	β	.9121	.8547	.9314	.9329	1.0674			
	x	.6327	1.2431	.3809	.4000	-.3755			
	X	.0916	.1007	.0804	.0622	-.0382			
	N_θ	6.9050	12.3450	4.7350	6.4280	9.8210			
4-8	β	.9906	.9815						
	x	.1044	.1837						
	X	.0594	.0384						
	N_θ	1.7560	4.7890						

Table 2. Computations from the seventh general observation period (MIT data only).

Time	Layer (m)	β	x	X	N_θ
1235	2-4	1.0562	-.9350	-.0924	11.118
1435	8-16	1.1041	-.5013	-.0518	9.669
"	4-8	1.0957	-.9463	-.0598	15.819
"	2-4	1.0050	-.9160	-.0924	9.909

Table 3. Computations from the sixth general observation period (JHU data only).

Time	Layer (m)	β	x	X	N_θ
1035	1.6-3.2	1.0394	-.2192	-.0365	5.999
1235	"	1.0657	-.3085	-.0493	6.262
1035	.8-1.6	1.0299	-.1247	-.0380	3.280
1435	1.6-3.2	1.0490	-.2979	-.0683	4.365

Table 4. Computations of the roughness parameter

Date	Time	z_0 (m) (MIT data only)
31 August	0635	0.017960
"	1835	0.012537
1 September	0635	0.015483
7 September	0635	0.174535
"	1835	0.013689
Date	**Time**	**z_0 (m) (JHU data only)**
31 August	0635	0.008249
"	1835	0.007335

A similar comparison made from the results of this investigation, shown in Figs. 2 - 6, indicate that the range of N_θ is several times as great as that shown by Priestley [9], or as calculated by Lettau [7] from the Great Plains data. Likewise, values of x computed in this paper ranged up to $\pm$ 1.0. Examination of Priestley's scatter diagram, Fig. 1, shows that his range of $x < 0$ is much smaller (closer to neutral conditions) than the points for which $x<0$ in Figs. 2 - 6. There is also an indication in Fig. 1 that his curve is making a bend in the vicinity of $x=0$, similar to that on the right side of Figs. 2, 3, and 4.

It was noticed by this investigator that the computation of x was quite sensitive to erroneous values of the roughness parameter. In the case of very rough surfaces, the vertical wind profile may be represented in the form

$$u = \frac{u_*}{k} \ln \frac{z-d}{z_0}$$

where d is called the datum-level displacement [3]. Lettau [7] makes use of the zero-point displacement $D=d + z_0$ in computing values of z_0 at the JHU site, and obtains an average value of $d=9.9$ cm. This means that the logarithmic profile produces a zero value of wind 9.9 cm above the zero point of nominal height, z_0. In the case of very dense vegetation, $d + z_0$ is equal to the height of the vegetation. Lettau [7] reports the value of d he obtains to be on the order of the grass height, 10 cm. Lake [5] , and Deacon [2] also make use of D in computing values

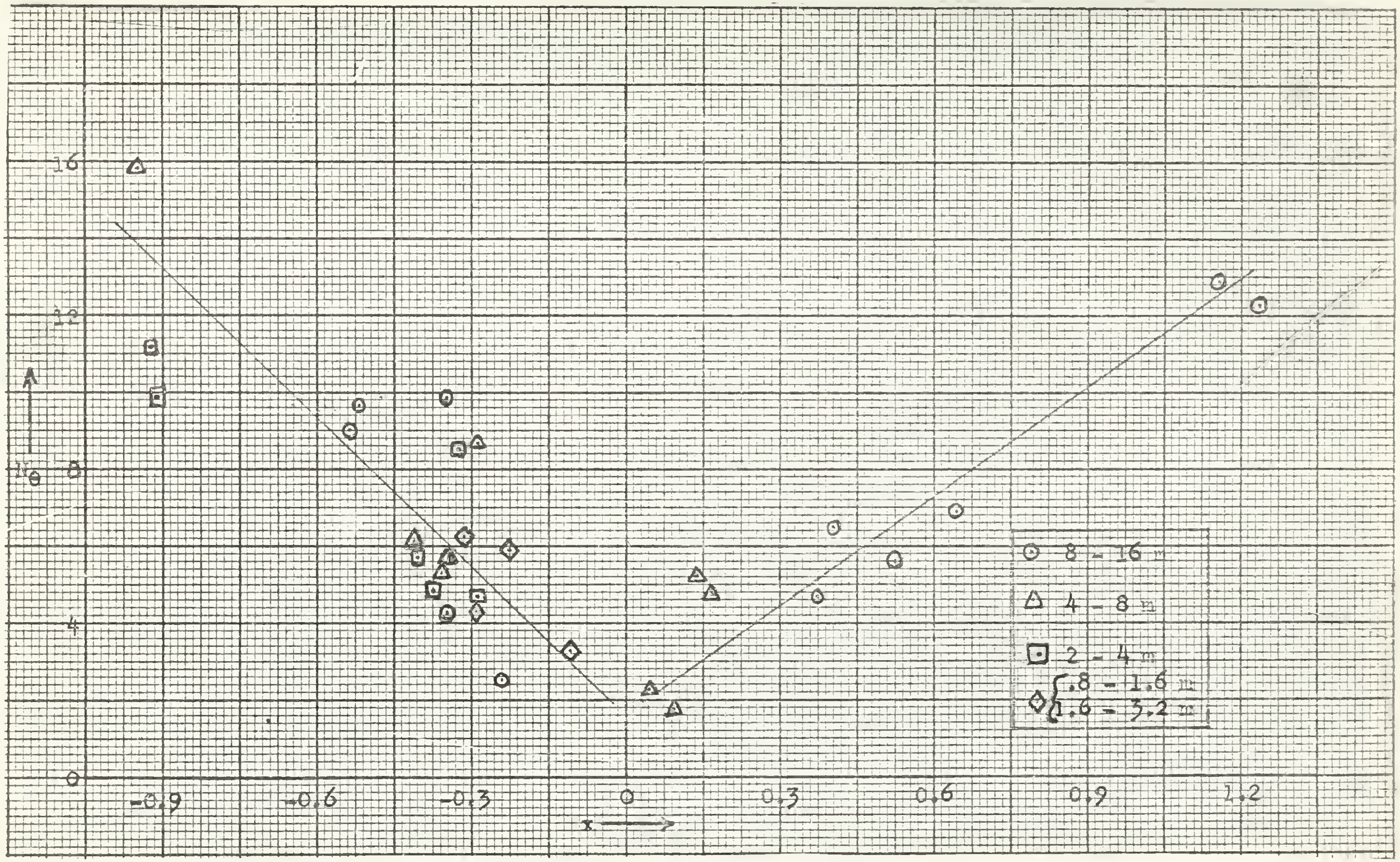

Figure 2. N_θ versus x for all computations.

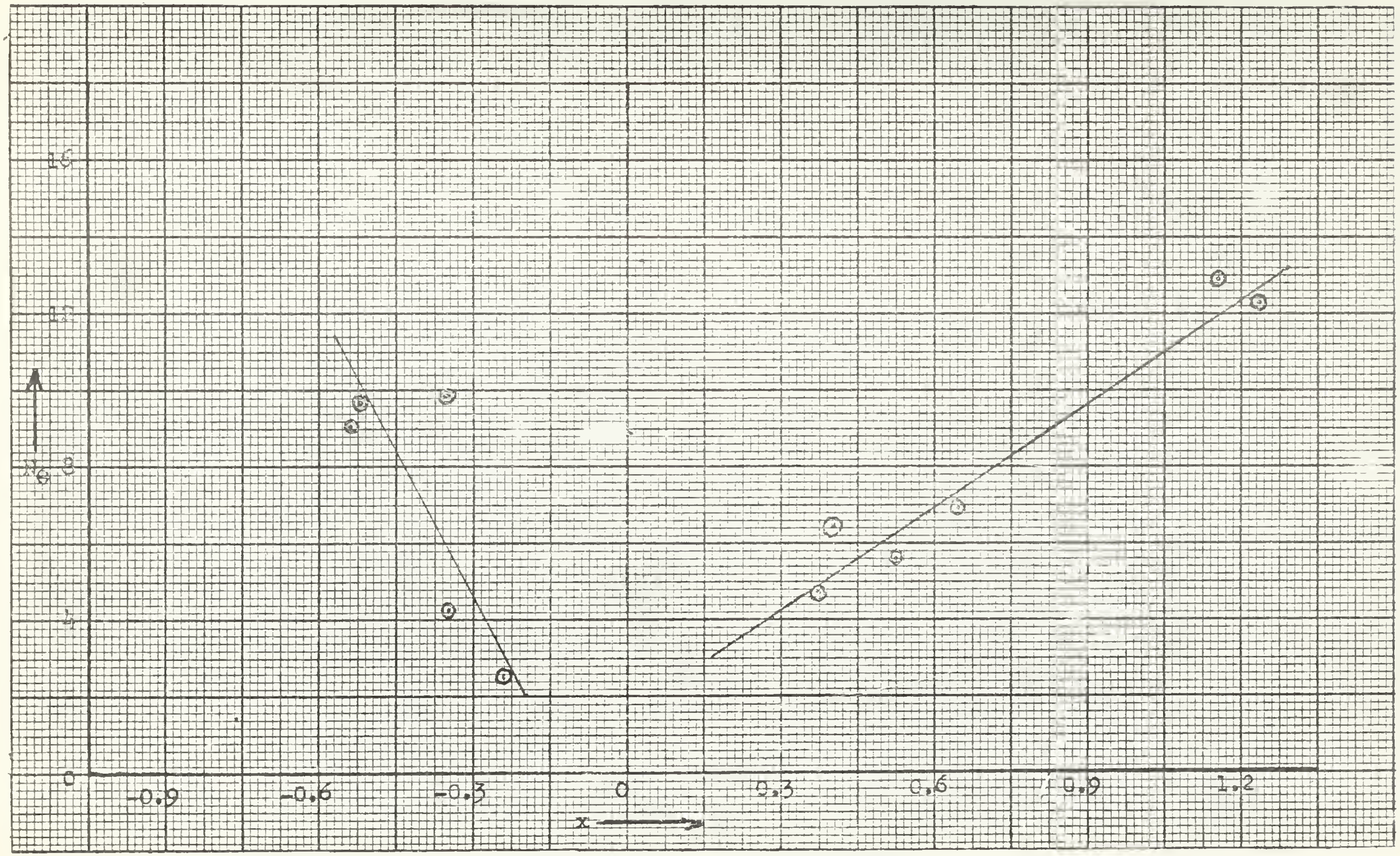

Figure 3. N_θ versus x for MIT data in the 8 - 16-m layer.

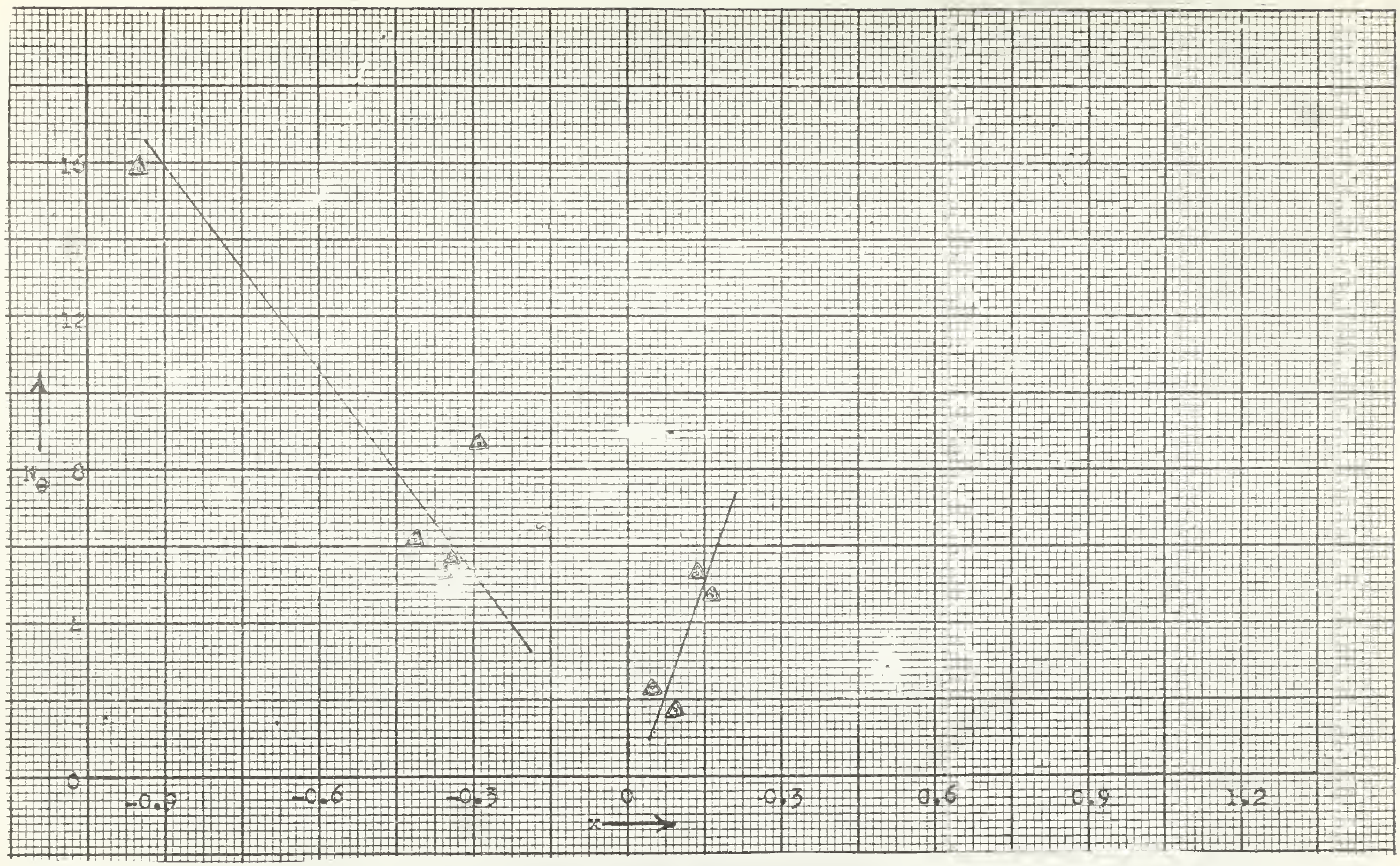

Figure 4. N_θ versus x for MIT data in the 4 - 8-m layer.

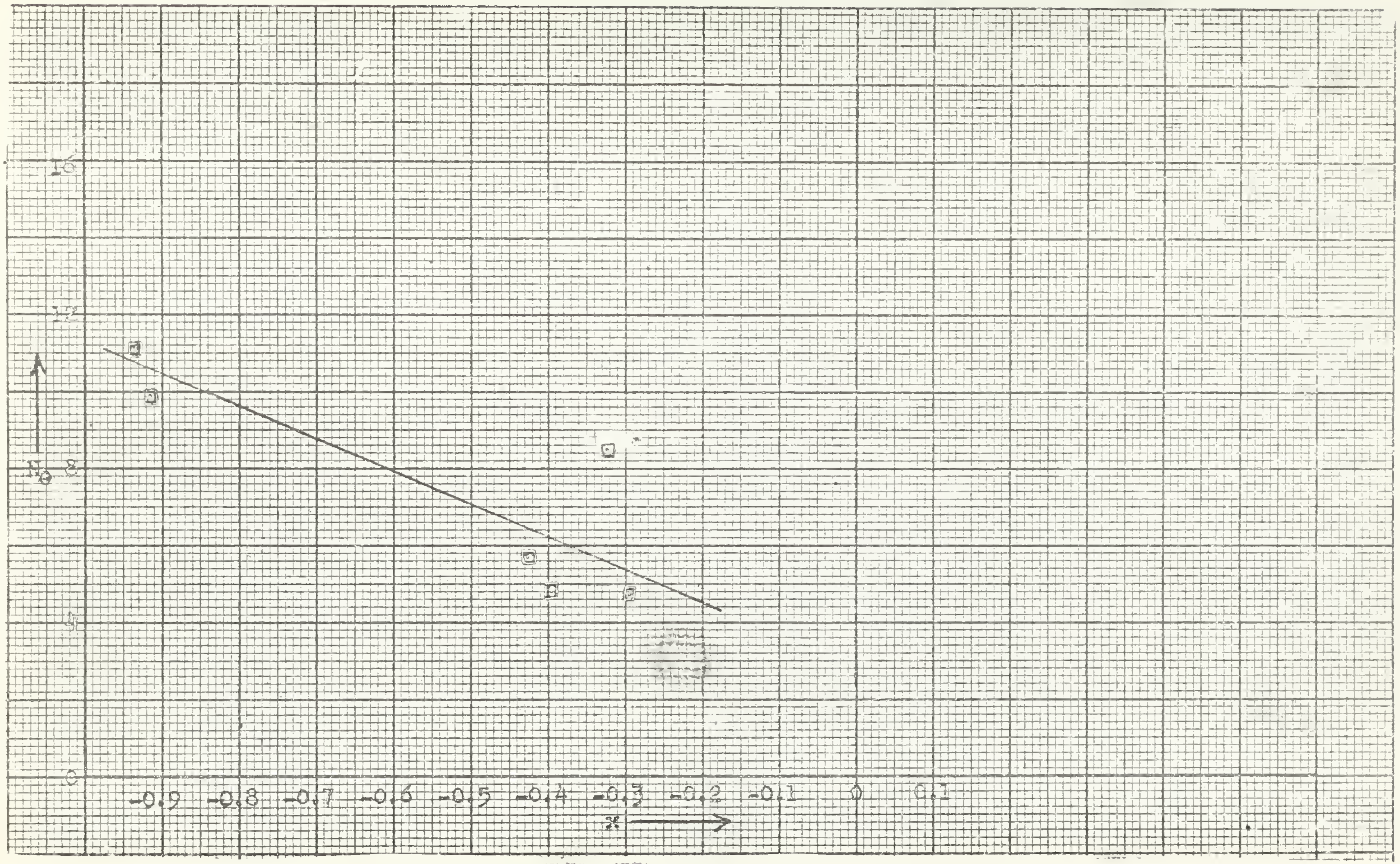

Figure 5. N_8 versus x for MIT data in the 2 - 4-m layer.

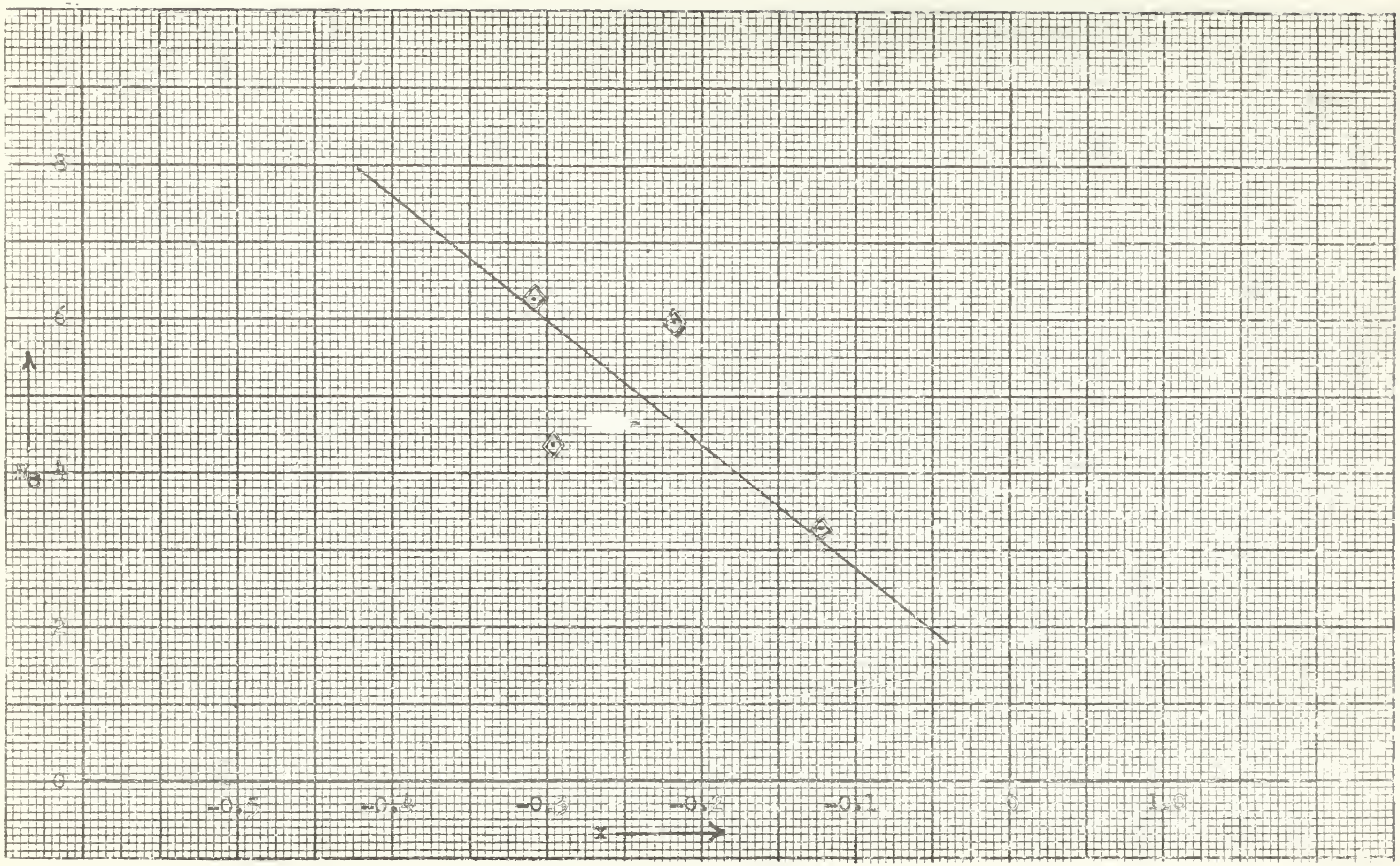

Figure 6. N_θ versus x for all JHU data.

of z_o. Deacon gives values of z_o for mown grass-surfaces of grass length 4.5 cm, ranging from $z_o = 2.4$ cm to $z_o = 1.7$ cm, depending upon the strength of the wind at the 2-m level. The results of this investigation gives values of z_o ranging from 0.73cm to 1.80cm, as shown in Table 4.

Lettau [7] calculated values of X using an equation similar to Eq. (38) at the 2,4, and 8-m levels. Values of X obtained in this work are consistent with his results and are valid at the geometric mean of the layers between 2, 4, 8, and 16 m.

Results of correlations of N_θ with x, X, and β, in the entire layer 2 m to 16 m are shown in Table 5.

Table 5. Correlations with N_θ with x, X, and β.

	Sample size	Correlation Coefficients N_θ vs. x	N_θ vs X	N_θ vs β	Significance Level
Stable	10	.965*	.707*	.911*	.639
Unstable	21	-.805*	-.130	-.244	-.440

Note: Significant correlations for the 0.05 level of belief are indicated by the asterisk (*).

The 5% significance level was derived from the test mentioned on pp. 124 of [4]. All of the correlations obtained under stable conditions were positive and significant, indicating that N_θ varies with stability in the same way with x, X, and β. However, the correlation was highest with x, which is to be expected since x explicitly contains N_θ in its formula. In unstable layers, all of the correlation coefficients were negative although only one of these was significant at the 5% level.

This particular correlation verifies, in a general way, that of Priestley [9], also taken under unstable conditions.

Fig. 7 shows a plot of N_θ versus time for each layer in which N_θ was determined. For the period 31 August - 1 September, the average values of N_θ are listed below.

Layer (m)	N_θ ave
8 - 16	7.5576
4 - 8	4.9735
2 - 4	5.9535

Note that Priestley's [9] mean value of N_θ at 1.5 m would be nearly one. However, he has values of N_θ as high as 4.0.

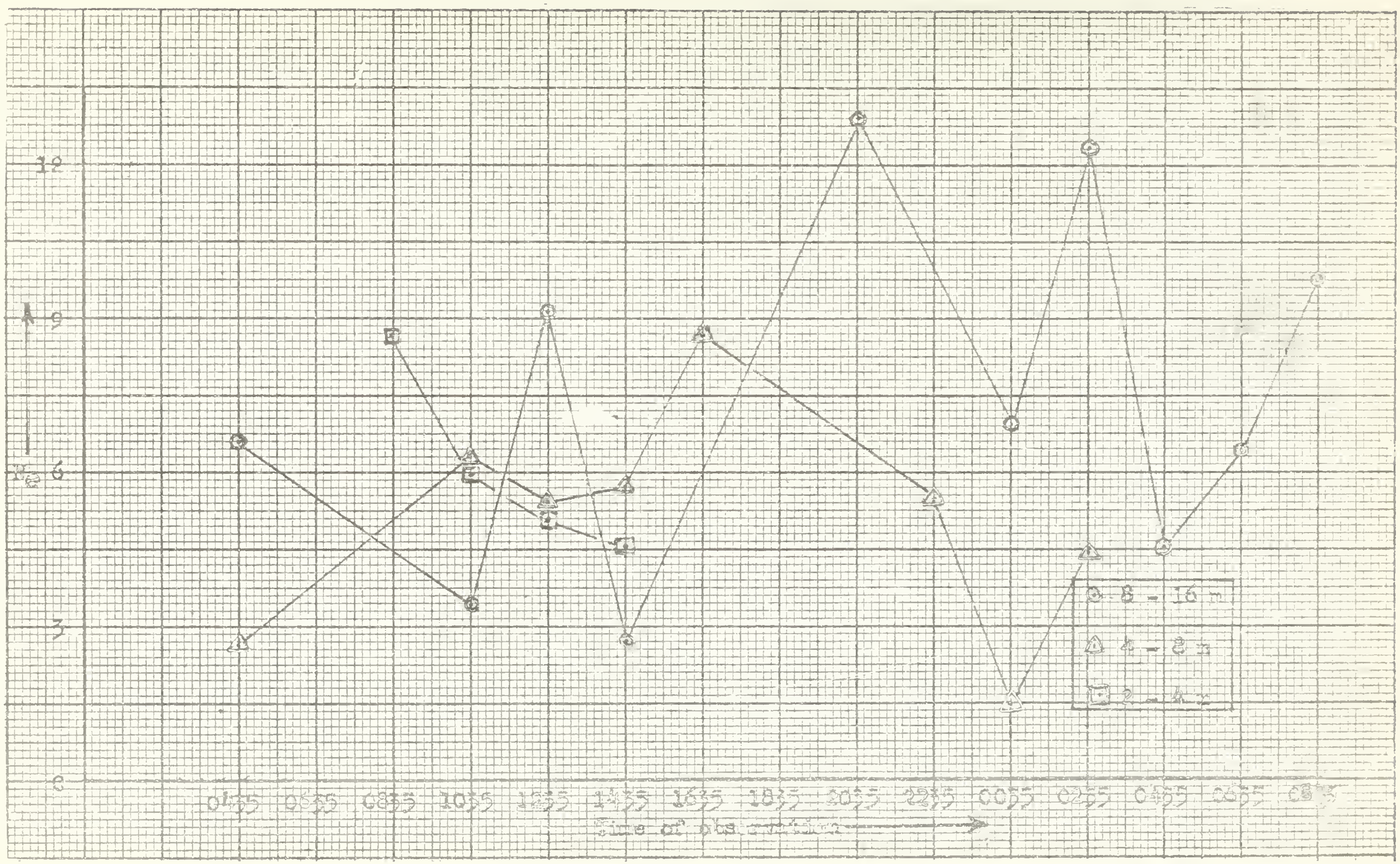

Figure 7. Plot of N_θ in the layers 8-16, 4-8, and 2-4 m, versus time, for the period 31 August - 1 September 1953.

6. Conclusions

The solutions obtained in this investigation are subject to rather stringent assumptions which undoubtedly account for some variance of the results from those of previous investigators. It has been shown in Section 5 that values of z_o, β, and X, lie in the range of previous investigations, but N_θ is somewhat larger than comparable values previously obtained.

The theory presented in this paper is developed to allow computations of the various parameters for non-adiabatic conditions, since the accuracy criteria of Section 4 ruled out all computations for near-neutral conditions. By Eq. (34), errors in ΔU_a which tend to increase x tend also at the same time to decrease X [from Eq. (39)] , and therefore increase N_θ.

However, this investigation did not attempt to verify the above observation, but to put forth a method of computing the pertinent turbulence parameters. The technique of representing $\overset{*}{\omega}_\theta/k$ by linear variation of the function Δu_a is a previously untested one and may be the contributing factor to the variations of N_θ thus obtained.

Since these computations ignored the use of a zero-point displacement in computing values of the roughness parameter, no conclusions as to the contribution of z_o to values of N_θ can be made. However, reasonable solutions for z_o were obtained.

The plot of N_θ versus x in Figs. 2 - 6 shows that N_θ increases, both with degree of stability and with degree of instability, and is near one for near neutral conditions. On

the basis of this paper it seems likely Fig. 1 would also show that N_θ increases with stability if more points to the right of the origin had been obtained.

The correlation of N_θ and x showed a higher degree of significance than that of N_θ versus X. This was to be expected as N_θ is explicitly contained in x. [See Eq. (21)]. Priestley [9] also reports that spuriously low correlations of N_θ and X can be expected as errors in measured gradients affect N_θ and X in an opposite sense.

Consistent with the approach of this investigation, Section 4 criteria ruled out solutions for many near-neutral layers. This investigation revealed that the seventh observation period (MIT data), and JHU data for the sixth observation period indicated near-neutral conditions much of the time; also solutions were unobtainable in the first five observation periods. Therefore, it can be concluded from this investigation that only the sixth observation period of the Great Plains Project contains data giving a significant range in stability, for the purposes of this research.

An inspection of Fig. 7 shows that N_θ is largest in the highest layer throughout most of the day. Secondly, N_θ increases with increasing departure from adiabatic conditions in either sense. This accounts for the JHU data giving fewer results at its lower observation levels.

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